

**REVIEW OF “M. H. ESCARDÓ: INFINITE SETS THAT  
SATISFY THE PRINCIPLE OF OMNISCIENCE IN ANY  
VARIETY OF CONSTRUCTIVE MATHEMATICS”**

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The omniscience principle for a set  $X$  says that, given any decidable property on the set, either there exists a member of  $X$  satisfying the property, or no member of  $X$  satisfies the property. This classically true disjunction is not acceptable constructively for already the case  $X = \mathbb{N}$ , known as the Limited Principle of Omniscience (LPO), would imply a constructive solution to the Halting Problem.

The intuitive reason why LPO should not be acceptable is that  $\mathbb{N}$  is infinite. In this, and a number of previous papers, the author presents infinite sets that even constructively satisfy the omniscience principle. The first such example is  $\mathbb{N}_\infty$ , the subset of non-increasing binary sequences of the Cantor space  $2^{\mathbb{N}}$ . More complex omniscient infinite sets exist as shown by a construction using infinite ordinals.

The method to prove a set omniscient goes via the notion of selection function. The paper contains a survey of results on selection functions, as well as detailed explanations on how the relevant constructions of Kreisel, Grilliot and Ishihara can be interpreted as statements about selection functions. Selection functions can, perhaps, also be seen as constructive versions of Hilbert’s  $\epsilon$ -operator.