REVIEW OF "F. ASCHIERI AND S. BERARDI: A NEW USE OF FRIEDMAN'S TRANSLATION: INTERACTIVE REALIZABILITY"

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Friedman-Dragalin's A-translation is a technique allowing to show the closure under Markov's Rule ($\mathrm{T} \vdash \neg\neg\exists x P(x)$ implies $\mathrm{T} \vdash \exists x P(x)$, for $P(x)$ a prime formula) for constructive theories such as the intuitionistic version of arithmetic in all finite types (HA^ω) or intuitionistic Zermelo-Fraenkel set theory.

In this paper, the authors show that their technique of *interactive realizability* can also be seen as a way to apply the A-translation, for the theory $\mathrm{HA}^\omega + \mathrm{EM}_1 + \mathrm{SK}_1$ extending intuitionistic higher-type arithmetic with the schema of excluded middle for Σ_1-predicates and Skolem axioms over decidable predicates:

$$(\mathrm{EM}_1) \qquad \forall xy(\exists z T(x,y,z) \vee \forall z \neg T(x,y,z))$$

$$(\mathrm{SK}_1[\phi]) \qquad \forall xyz(T(x,y,z) \to T(x,y,\phi(x,y))),$$

where $T(x,y,z)$ denotes Kleene's T-predicate on $\mathbb{N}^3$, and ϕ is a constant symbol of type $\mathbb{N}^2 \to \mathbb{N}$.

More precisely, denoting by B^A the A-translation of formula B using formula A, the authors' aim is to show

$$\mathrm{HA}^\omega \vdash \mathrm{EM}_1^{\mathcal{A}(s)} \text{ and } \mathrm{HA}^\omega \vdash \mathrm{SK}_1[s]^{\mathcal{A}(s)}$$

in Theorem 2, for $\mathcal{A}(s)$ defined by

$$\exists xyz(T(x,y,z) \wedge \neg T(x,y,s(x,y))),$$

and then conclude in Theorem 3 that from a proof of $\forall x \exists y P(x,y)$ in $\mathrm{HA}^\omega + \mathrm{EM}_1 + \mathrm{SK}_1$, P-prime, one can get a proof of $(\forall x \exists y P(x,y)^{\mathcal{A}(s)})$ in $\mathrm{HA}^\omega + \mathrm{EM}_1 + \mathrm{SK}_1$, and hence a proof of only $\forall x \exists y P(x,y)$ in HA^ω.

The larger part of the paper (sections 3 and 4) is dedicated to giving a streamlined version of the technique of interactive realizability, with a number of proofs referring to the PhD thesis of the first author. Section 5 is dedicated to a comparison between Kreisel's no-counterexample interpretation and interactive realizability. Finally, in Section 6, Theorem 3 (Theorem 27) is established.

It remains unclear to the reviewer what (meta-)theory can be used to establish the soundness of the interactive realizability interpretation (ex. Theorem 22), something that is standard for modern formulations of realizability techniques like Kreisel's modified realizability or Gdel's Dialectica interpretaion.

Date: November 1, 2015.