REVIEW OF "A SEMANTIC APPROACH TO CONSERVATIVITY" BY TOMASZ POŁACIK, STUDIA LOGICA (2016) 104: 235-248 (MR3477360)

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This paper presents a study of conservativity of classical theories T over their intuitionistic versions, from a semantic point of view: instead of studying directly the proof transformation

$$\mathrm{T} \vdash^c A \quad \Longrightarrow \quad \mathrm{T} \vdash^i A,$$

one studies the contrapositive statement

$$\mathrm{T} \nvdash^i A \quad \Longrightarrow \quad \mathrm{T} \nvdash^c A,$$

which by completeness amounts to the statement

$$\exists \mathcal{M}(M \nVdash T \supset A) \quad \Longrightarrow \quad \exists \mathcal{M}'(M' \nvDash T \supset A).$$

The difficulty is then to find classes of Kripke models $\mathcal{M}$ and classes of formulas A for which this statement holds.

The paper presents two separate cases when results along these lines can be obtained, Theorem 4.10 on T-normal models and the class of formulas $\mathcal{A}$, and Theorem 5.7 involving the Friedman translation and conversely well-founded Kripke models with constant domains.

Finally, a new application, previously not available by proof theoretic methods is shown, the conservativity of ZF over CZF (Friedman showed conservativity over IZF).

The meta-theory used is classical, with forcing of implication being defined by a disjunction classically equivalent to an implication. This is not a great 'sin', since, according to the current state-of-the-art, the completeness of intuitionistic logic with respect to Kripke models requires classical logic ad the meta-level anyway.