(MR3350402) REVIEW OF "SEMANTICAL COMPLETENESS OF FIRST-ORDER PREDICATE LOGIC AND THE WEAK FAN THEOREM" BY VICTOR N. KRIVTSOV (2014)

DANKO ILIK

This paper shows the equivalence between a version of the Fan principle and completeness theorems for intuitionistic and classical first-order and propositional logic, over an intuitionistic meta-theory WKVS, when the language of formulas is countable, and when validity is Σ_1-definable.

More precisely, taking WKVS to be the language of WKV (Heyting arithmetic with quantification over number-theoretic functions) extended with relation symbols for species—allowing the scheme of induction, unique choice, and extensionality over all formulas of the extended language, and admitting a comprehension principle only for formulas of the language of WKV—one can establish equivalence between the Fan principle,

$$\forall\alpha\exists n(\beta(\overline{\alpha}n) = 1) \to \exists m\forall\alpha\exists n(n \leq m \land \beta(\overline{\alpha}n) = 1),$$

(where $m, n \in \mathbb{N}$, $\alpha, \beta : \mathbb{N} \to \{0, 1\}$, and $\overline{\alpha}n$ denotes the finite initial segment of α with length n), and completeness of intuitionistic and classical first-order and propositional logic with respect to Beth models and "intuitionistic structures" (i.e. intuitionistic "Tarski" semantics)—for the restriction of validity in both kind of semantics to Σ_1-definable formulas, i.e. the existence of a decidable predicate P on $\mathbb{N}^3$ such that $k \Vdash A \leftrightarrow \exists nP(k, \ulcorner A\urcorner, n)$, respectively P on $\mathbb{N}^2$ such that $\mathfrak{M} \vDash A \leftrightarrow \exists nP(\ulcorner A\urcorner, n)$. This equivalence for completeness theorems is stated in Theorem 5.3, while Corollary 5.7 states the analogous equivalence for compactness theorems.

Date: November 1, 2015.