(MR3204975) Review of "Formal Baire space in constructive set theory" by Giovanni Curi and Michael Rathjen (2012)

Danko Ilik

March 5, 2015

Locales, or formal spaces, of constructive, point-free topology are defined by covering systems that arise from set-theoretic inductive definitions. When one works with a set theory much weaker than ZF, like the constructive system CZF, the question appears whether inductive definitions still suffice to prove the existence of certain formal spaces as sets. While this is the case for the Cantor space and the real line, CZF is not strong enough to prove that Baire space is a formal space, even if CZF is extended with the axioms of Countable Choice, Dependent Choices, and the Presentation Axiom. This is the main new result of the paper (Theorem 3.4). As a consequence one gets that the same system does not prove that the Brouwer (constructive) ordinals form a set.

Although technically precise, the article is readable to non-experts and self-contained. The Regular Extension Axiom (REA) is described which, when added to CZF, suffices to prove Baire space is a formal space (CZF+REA is a subsystem of ZFC but not of ZF). Connections to the important principles of monotone and decidable Bar Induction and the Fan Theorem are also made.